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The Use of 25-Grid Point of 3D Biharmonic Equation in Human Fac Recognition

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Abstract

The paper addressed model of human face images using Mathematical methods. Human face recognition becomes a major issue and has occupied an active area in many fields. It has strong links to the general area of pattern recognition and it is demanded in commercial and security applications as they are subject to biometric systems.

The basic problem in face recognition is the difficulty in solving the fourth - order Biharmonic Equation numerically to generate an Elliptic Surface. It's also not easy to combine the Elliptic Surface discrete quantities with human face images.

The study concerned with the efficiency of solving the three dimensional Biharmonic Equation in the field of human face recognition along with the modeling of face images in order to identify human by facial information. The process has involved the division of the surface by using forward finite differences method in order to get the coefficient matrices of the grid points which represent the elliptic surface and then taking the human face images by using 3D Camera and from data base. The images then have been used to get the statistic data information and the plotting curves of these data by using MATLAB.

It has been found that the 25-grid point's surface is conservative field.

Keyword: Human face recognition, fourth - order Biharmonic Equation, Finite Difference,

1.Introduction

In the modern life, human crime increases daily and takes many different shapes. Therefore, the need for personal security and access control is becoming an important issue. Biometrics is the technology which is expected to replace traditional authentication methods; that could easily be stolen, forgotten and duplicated. Fingerprint, face, iris and voice prints are commonly used biometric features. Among these features, face provides a more direct, friendly and convenient identification method and more acceptable approach compared to the individual identification of other biometric features. Hence, face recognition is one of most important part in biometrics [1]. Biometric signatures enable automatic identification of a person based on physiological or behavioral characteristics, Physiological biometrics are biological or chemical traits that are innate or naturally grown, while behavioral biometrics are mannerisms or traits that are learned or acquired. Biometrics technologies are becoming the foundations of an extensive array of highly secure identification and personal verification solutions. Compared with conventional identification and verification methods based on Personal Identification Numbers (PINs) or passwords, biometrics technologies offer some unique advantages. First, biometrics are individualized traits while passwords may be used or stolen by someone other than the authorized user. Also, a biometric is very convenient since there is nothing to carry or remember. In addition, biometric technology is becoming more accurate and inexpensive. Among all biometrics, face biometric is unique because face is the only biometric belonging to both physiological and behavioral categories [6].

Computer analysis of face images deals with a visual signal that is registered by a digital sensor as an array of pixel values. The pixels may encode color or only intensity; the pixel array can be represented as a point (i.e., vector) in a $m \times n$ dimensional image space by simply writing its pixel values in a fixed order [2]. The principle energy source for images in use today is the electromagnetic energy spectrum [3,4].

Face recognition has a significant and difficult task to identify a person by his own facial information; thus geometry of human face is extremely complicated issue.

2. Digital Image Representation

Any image may be defined as a two-dimensional function $f(x, y)$ where x and y are spatial (plane) coordinates and the amplitude of f at any pair of coordinates (x, y) is called the intensity of the image at the points. The term gray level is used often to refer to intensity of monochrome image.

Color images are formed by combination of individual 2-D images, for example. In the R G B color system, a color image consist of three (red, green, and blue) individual component images , for this reason , many technique developed for monochrome image can be extended to color images by processing the three component images individually. Converting such an image to digital form requires that the coordinates as well as the amplitude be digitized. Digitizing the coordinate values is called sampling, digitizing the amplitude values is called quantization. Thus, when x, y and the amplitude values of f are all finite quantities, we call the image a digital image [3,4].

3. The idea of geometrical shape and PDEs:

In the last three decade Mathematicians joined the field of face recognition; they used partial differential equations PDEs to generate geometrical shape. The subject of Partial differential Equations (PDEs) emerged in 18th century as Ordinary Differential Equations (ODEs) failed to describe some physical phenomena. Since then, many physical phenomena and paramount discoveries have been branded with the PDEs. Geometric modeling using PDEs has been widely studied in computer graphics[5].

Human face image is so complicated shape, and partial differential equations are regarded as a suitable frame to deal with the complex shapes. On the other hand, Biharmonic Equation (BE) is an elliptic equation, so be generates an elliptic surface and frame of the most human faces which have elliptic frame, so long the researcher is investigating the elliptic frame and since human faces have the same shape, this indicates the fact that the researcher is working in similar framework. Hence the human face information will be inserted elliptic surface which has been generated from (BE) and the it would as a container.

There are two methods to solve (BE), analytically and numerically. In analytical method (BE) already solved by Eyad Elyan, Hassan Ugail and used in human face recognition problems.

Identification or verification of human by his/her own face information is a very difficult task; to represent and model human face geometric mathematically is an intricate task.

In this study the researcher going to examine the ability of Elliptic Surface based on 3D (BE) in human face recognitions field, using Mathematical divide methods, to convert the continuous function (surface of (BE)) into discrete quantities matrices in order to generate grid point surface and the surface will divide using partial Differential Equations treatment using Finite Difference with suitable boundary conditions.

in order to read human face image properly, one has to place it on a suitable format for poke to converting electromagnetic energy source (image) to digital number (discrete quantities) which represents the target face in digital form.

Two discrete quantities (grid point and digital image) will combine with the aim of generating new intensity image with statistical information; this information is unique for individual image according to different skin surfaces[8].

4.N umerical Solution of 3D Biharmonic Equation:

A numerical solution of a partial differential equation (PDE) involves a discretization procedure by which the continuous equation is replaced by a discrete algebraic equation. The discretization procedure consists of an approximation of the derivatives in the existing PDE by differences of the dependent variables, which are computed only at discrete points (grid or mesh points).The discretization of the continuous problem inevitably introduces an error in computing the derivatives and, as a result, an error in the computational solution may occur. In general we can describe the relation between PDE and discrtization procedure for developing a finite-difference equation (FDE)as a linear relation between discrete values of the unknown function computed on grid point using Taylor series expansion PDE can be rewritten in the following form:

PDE $=$ FDE $+$ TE, where the remainder, TE, is the truncation error. The truncation error (TE) is the difference between the original PDE and itsFDE approximation.It is expressed in terms of the Taylor series expansion and its lowest-order term gives the accuracy order of the discretization method applied to the original PDE. It should be noted that one can estimate the numerical solution error for a finite-difference representation as the order of the leading term in the remainder (TE), which can be considered as a close approximation to the error, provided that the grid size is small. However, the complete evaluation of the numerical solution error must be based on comparison with the exact solution [7].

To get standard difference schemes for the (BE) based on 25 – points stencils with the fourth-order accuracy, the researcher extend the idea of (BE) in 2D which mentioned in [7] to 3D as follow.

The computational domain of G, let G be the Elliptic surface in 3D, such that ,

$G = \{0 \leq x \leq l_x, 0 \leq y \leq l_y, 0 \leq z \leq l_z\}$be a three dimensional computational domain.Suppose three discrete equally spaced sets, w_x, $\mathrm{w_y}$, and $\mathrm{w_z}$,of points be defined as general by

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$w_x = \{x(i) = ih\_x, 0,1 \ \ldots\ldots M, Mh_x = l_x\}$ (1)
$w_y = \{y(j) = jh_y, j = 0,1 \ \ldots\ldots N, Nh_Y = l_y\}$ (2)
$w_z = \{z(k) = kh_z, k = 0,1 \ \ldots\ldots S, Sh_z = l_Z\}$ (3)
Computational grid Ω. Let a computational gridΩ on the domain G be a set of points defined by

$$\Omega = w_x \times w_y \times w_z$$
$$= \{w_{i,j,k} = (ih_x, jh_y, kh_z), i = 0,1, \ldots\ldots.. M, j = 0,1, \ldots\ldots N, k = 0,1, \ldots\ldots.. S\}$$

Each (i, j, k)th node of the grid points Ω refer to a point with $x(i), y(j)$, and $z(k)$ coordinates defined by equations (1) , (2) and (3) respectively. In this case the grid Ω is equally spaced in the x, y ,andz-directions. But generally $h_x \neq h_y \neq h_z$

Let the function continuous $f(x, y)$ defined in domain G and the discrete function $f_{i,j,k}$= $f(x_i, y_j, z_k)$ where
$x_i = x(i), \in w_x, y_i = y(j) \in w_y$ and z_k=$z(k)$ is a grid point representation of $f(x_i, y_j, z_k)$ on the
Computational gridΩ.
The discritzation central node (i, j, k) of 25-grid point of 3D biharmoinc equation are defined by

$$\sigma_{i,j,k}{}^{(25)} = \{w_{p,q,g}, p = i-2, i-1, i, i+1, i+2, q = j-2, j-1, j, j+1, j+2, g = k-2, k-1, k, k+1, k+2\}$$

Where $w_{p,q,g} \in \Omega$
Taylor series expansion on$\sigma_{i,j,k}{}^{(25)}$ of discrete biharmonic equation of $f_{i,j,k}$ on central (i, j, k) node describe as follow:

$$f_{i\pm2,j\pm2,k\pm2} = f_{i,j,k} + \sum_{m=1}^{\infty} \frac{1}{m!}(\pm h_x D_x \pm h_y D_y \pm h_z D_z)^m f_{i,j,k} \quad (4)$$

D_x,$D_y$and $D_z$are operator of partial derivatives.
The general form fourth order 3Dbiharmonic equation gives as follows.

$$(\nabla^2)^2 = \frac{\partial^4 U}{\partial x^4} + \frac{\partial^4 U}{\partial y^4} + \frac{\partial^4 U}{\partial z^4} + 2\frac{\partial^4 U}{\partial x^2 \partial y^2} + 2\frac{\partial^4 U}{\partial x^2 \partial z^2} + 2\frac{\partial^4 U}{\partial y^2 \partial z^2} = 0 \quad (5)$$

Using finite difference and Taylor series expansion we get the approximate solution of (5)as follow.

$$42U(x_i, y_j, z_k) - 12[U(x_{i+h}, y_j, z_k) + U(x_i, y_{j+h}, z_k) + U(x_i, y_j, z_{k+h}) +$$
$$U(x_{i-h}, y_j, z_k) + U(x_i, y_{j-h}, z_k) + U(x_i, y_j, z_{k-h}) + U(x_{i+2h}, y_j, z_k) +$$
$$U(x_i, y_{j+2h}, z_k) + U(x_i, y_j, z_{k+2h}) + U(x_{i-2h}, y_j, z_k) +$$
$$U(x_i, y_{j-2h}, z_k) + U(x_i, y_j, z_{k-2h})])] + 2[U(x_{i+h}, y_j, z_{k+h}) +$$
$$U(x_{i+h}, y_{j+h}, z_k) + U(x_i, y_{j+h}, z_{k+h}) + U(x_{i-h}, y_j, z_{k+h}) +$$

$$U(x_{i-h}, y_{j+h}, z_k) + U(x_{i+h}, y_{j-h}, z_k) + U(x_{i+h}, y_j, z_{k-h}) +$$
$$U(x_i, y_{j-h}, z_{k+h}) + U(x_i, y_{j+h}, z_{k-h}) + U(x_{i-h}, y_{j-h}, z_k) +$$
$$U(x_{i-h}, y_j, z_{k-h}) + U(x_{i-h}, y_{j-h}, z_k)] = 0 \quad (6)$$

THE coefficient matrix of 3D (BE) of fourth order from equation (6) represented in equation (7)

$$\text{25-Grid points matrix } A = \begin{bmatrix} 2 & 2 & -12 & 2 & 2 \\ 2 & -12 & -12 & -12 & 2 \\ -12 & -12 & 42 & -12 & -12 \\ 2 & -12 & -12 & -12 & 2 \\ 2 & 2 & -12 & 2 & 2 \end{bmatrix} \quad (7)$$

5. Results:

In order to examine the idea and satisfy this experiment, in this stage we use MATLAB language program with image processing tool. As mentioned above, the proposed idea has been implemented using MATLAB and tested using 300 -images of faces, most of images are getting from website of data base and a few of them acquired by 3D camera 14 megapixel**.**

In order to satisfy the idea, the 25-grid point of matrix A combined

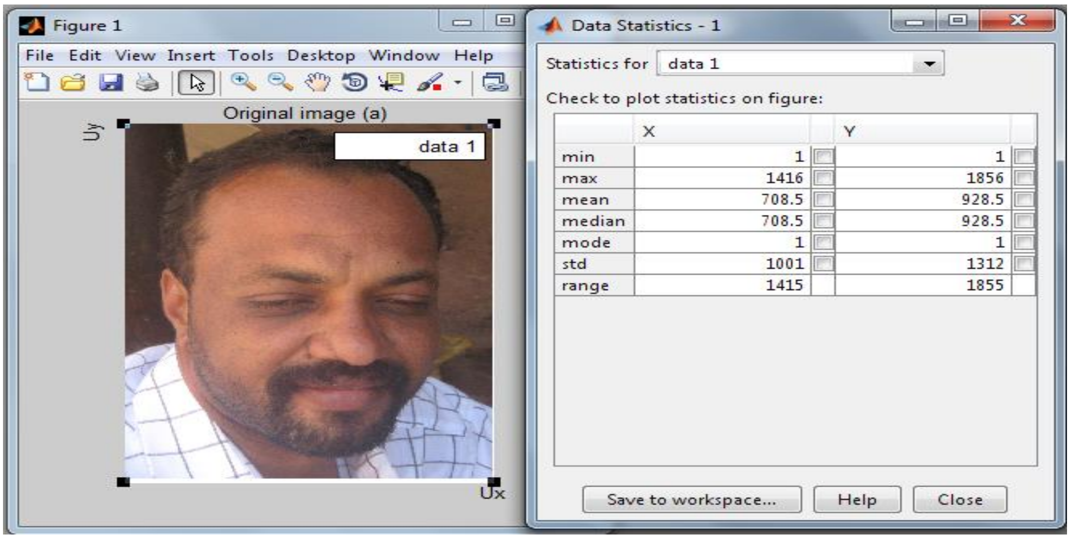

Figure (1) show the original image (a) and its statistic data

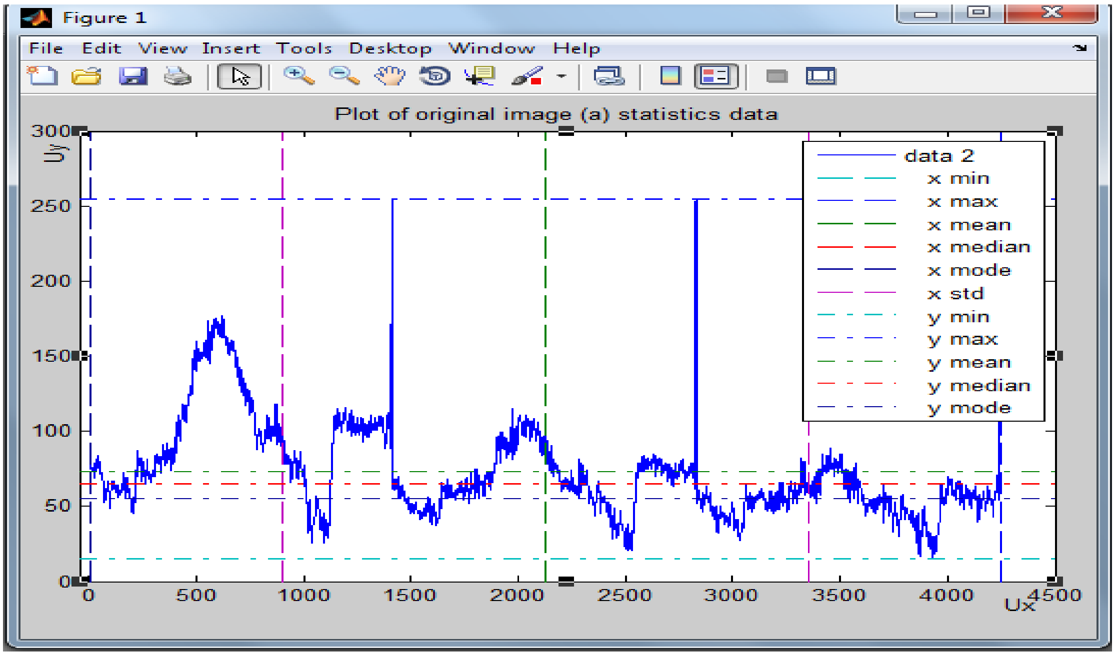

Figure (2) show the plot of original image (a) and its statistic data

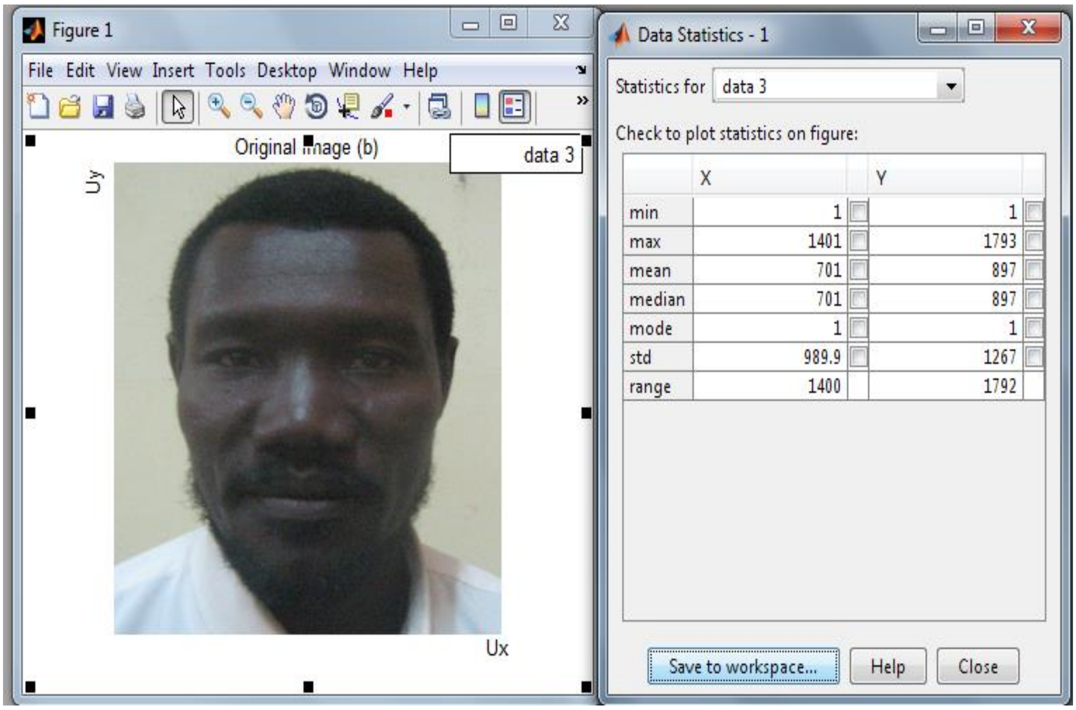

Figure (3) show the original image (b) and its statistic data

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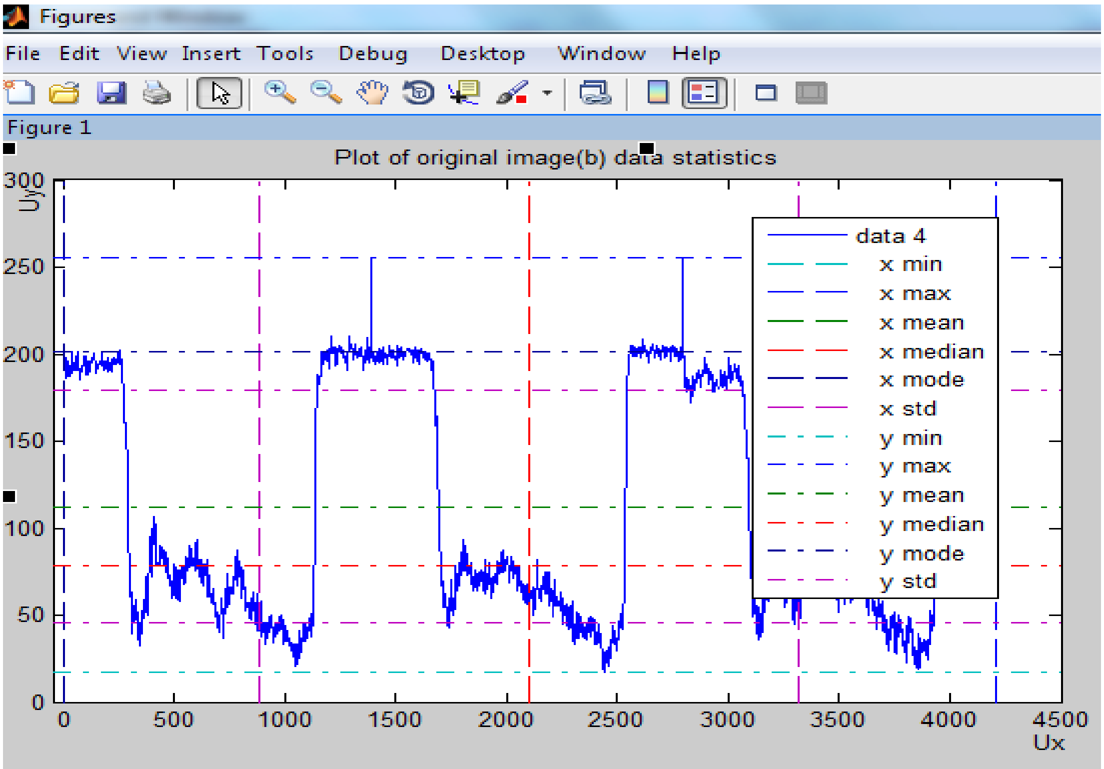

Figure (4) show the plot of original image (b) and its statistic data

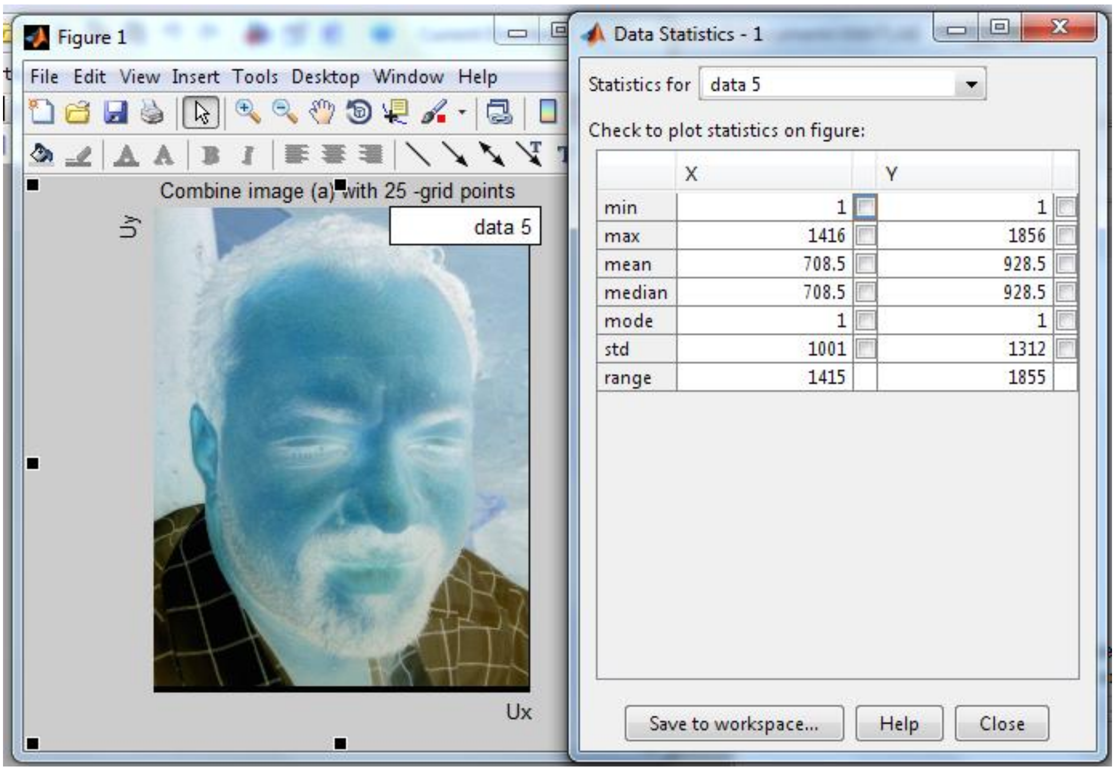

Figure (5) show the combine original image (a) with 250grid points

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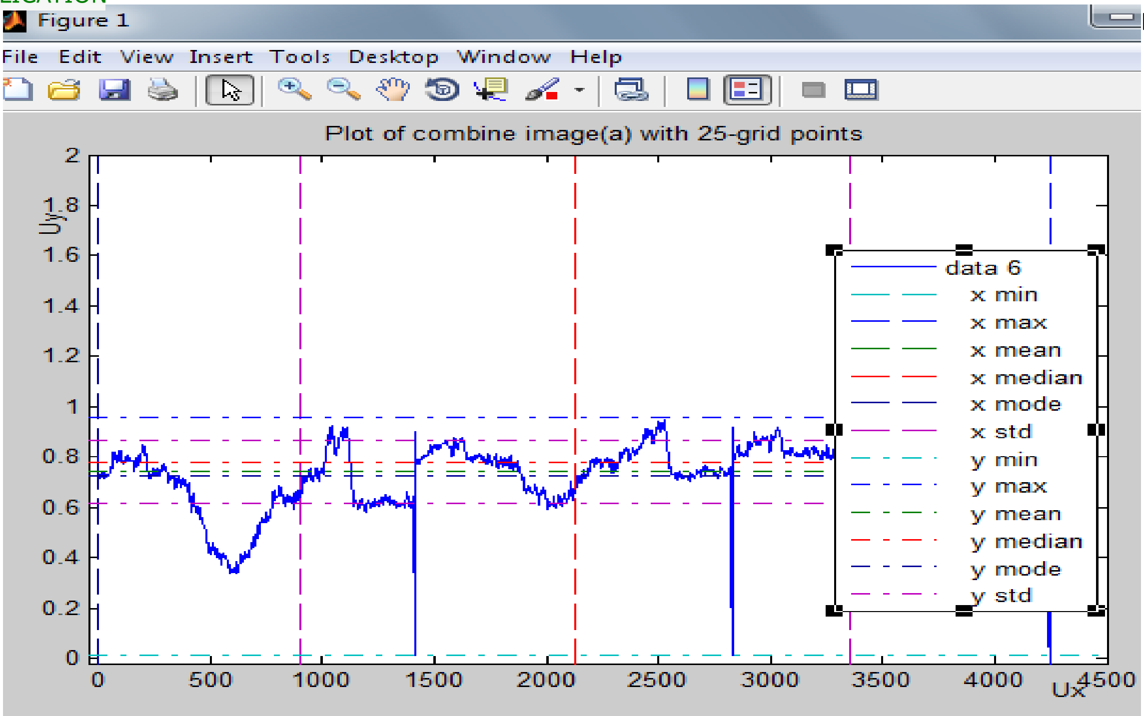

Figure (6) show the plot of combine original image (a) with 250-grid points

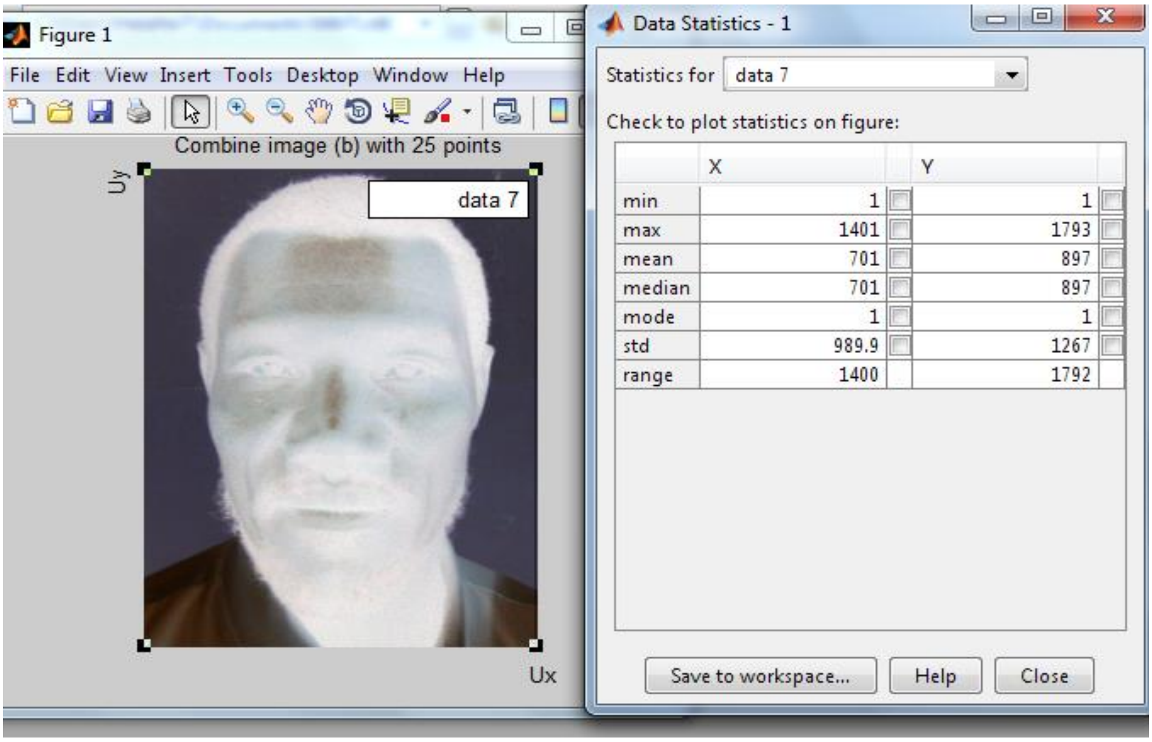

Figure (7) show the combine original image (a) with 250grid points

ISSN: 2208-2093

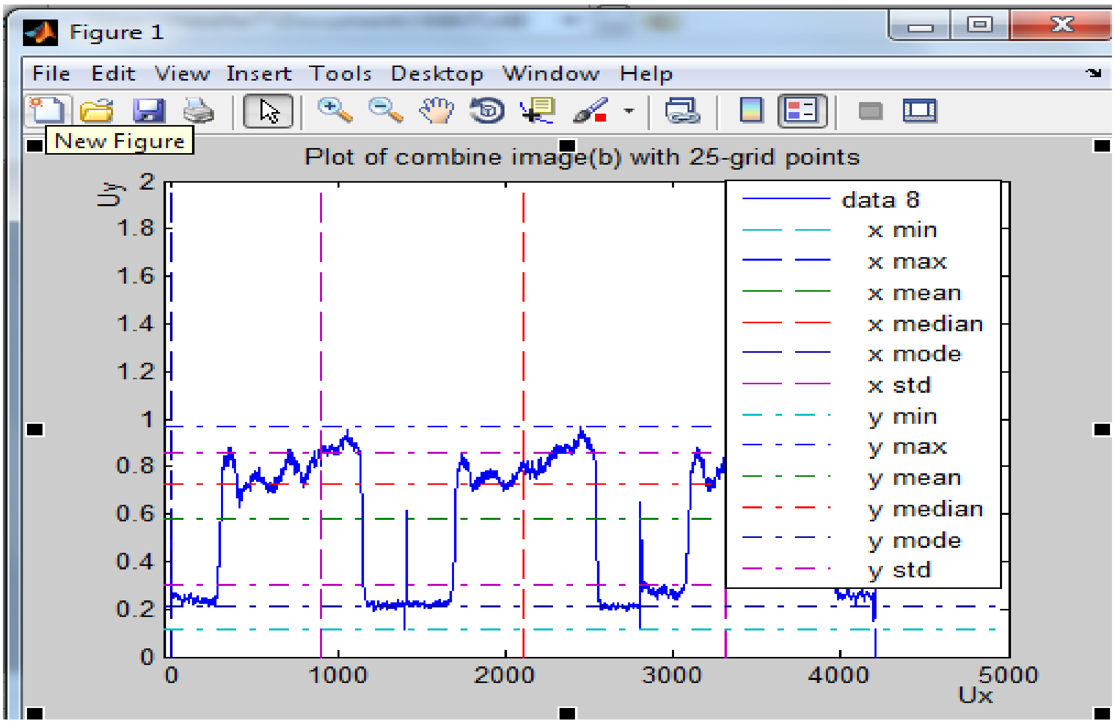

Figure (8) show the plot of combine original image (a) with 250grid points.

The results shows, the 25-grid points of elliptic surface of (BE) is conservative field, such that the images and its statistic information for both original and combined images are identical. Also the plotted curve of original and combined images, even the appeared images are closely similar except the combine images are in cloud intensity gray levels.

The experimental results of 300 human face images in this study shows that any face image has unique statistics data information according to different skin colors and properties. Therefore we use this fact to identify the face between many, using one –to many searches technique, beside the images match.

6. Conclusion:

In this work. The numerical divide technique has been used to convert the continuous function to discrete quantities, and the continuous images have been converted to digital in order to evaluate the efficiency of Fourth-order Biharmonic Equation in field of human face recognitions using MATLAB language program.

It found that the intensity pixels number of any face is unique consistent with skin colors.

Further work on this theme we need to develop algorithms to deal with images data comparison, to decide whether the method is applicable or not in investigation purpose.

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